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The Algebras with Discrete Derived Category

Dieter Vossieck

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We classify the finite-dimensional algebras over an algebraically closed field whose bounded derived category does not admit an infinite continuous family of pairwise non-isomorphic indecomposable complexes. © 2001 Academic Press

INTRODUCTION

In representation theory, an important role is played by the bounded derived categories $\mathscr{D}^b(k[Q])$ of finite-dimensional quiver-algebras over an algebraically closed field [H1]. If Q is Dynkin, $\mathscr{D}^b(k[Q])$ has only finitely many indecomposables up to shift and isomorphism. If Q is extended Dynkin, there are infinite "continuous" families of non-isomorphic indecomposables in $\mathscr{D}^b(k[Q])$.

Outside the class of algebras having the same derived category as a quiver-algebra, some kind of intermediate behaviour is possible. For example, if $k[\epsilon]$, $\epsilon^2 = 0$, is the algebra of dual numbers, representatives of the indecomposables of $\mathscr{D}^b(k[\epsilon])$ are the complexes $\cdots \to 0 \to k[\epsilon] \xrightarrow{\epsilon} \cdots \xrightarrow{\epsilon} k[\epsilon] \to 0 \to \cdots$ and $\cdots \to 0 \to k \to 0 \to \cdots$; even up to shift, there are infinitely many of them, but they do not form "continuous" families.

In the present paper, we prove that the only algebras showing the same behaviour are certain gentle algebras. To complete the picture (and for the benefit of the proof of our result), we also look at categories of perfect complexes and repetitive algebras.

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1. DERIVED CATEGORIES AND REPETITIVE SPECTROIDS

We fix an algebraically closed base field k and replace finite-dimensional algebras by locally bounded spectroids [GR, 8.3], which will be denoted by A, $B,\dots$.

1.1

We denote by $\mathscr{D}^b(A)$ the derived category of bounded complexes of mod A, the category of finite-dimensional (left) A-modules [V, H1]. With $X \in \mathscr{D}^b(A)$ we associate the sequence $\underline{\mathrm{Dim}}\,\mathrm{X} := (\underline{\dim}\, H^i(X))_{i\in\mathbf{Z}}$, where $\underline{\dim}\, H^i(X)$ is the dimension vector of the cohomology module $H^i(X)$; i.e., its class in the Grothendieck group $K_0(A)$ of mod A. If $X \to Y \to Z \to X[1]$ is a (distinguished) triangle of $\mathscr{D}^b(A)$, then $\underline{\mathrm{Dim}}\, Y \le \underline{\mathrm{Dim}}\, X + \underline{\mathrm{Dim}}\, Z$, as the corresponding long exact cohomology sequence shows.

If $F : \mathscr{D}^b(A) \to \mathscr{D}^b(B)$ is an exact functor and $X \in \mathscr{D}^b(A)$, $\underline{\mathrm{Dim}}\, F(X)$ does not depend solely on $\underline{\mathrm{Dim}}\, X$. We have, however, the (sharp) estimate

$$\underline{\mathrm{Dim}}\, F(X) \le \sum_{i\in\mathbf{Z}} \sum_{a\in A} \dim H^i(X)(a) \underline{\mathrm{Dim}}\, F(S_a[-i]),$$

where S_a is the simple supported by the object $a \in A$. Indeed, the truncations

$$\tau_{\le i} X = \left(\to X^{i-2} \to X^{i-1} \to \mathrm{Ker}(d^i) \to 0 \to 0 \to\right)$$

give rise to triangles

$$\tau_{\le i-1} X \to \tau_{\le i} X \to H^i(X)[-i] \to (\tau_{\le i-1} X)[1],$$

and X can be viewed as an iterated extension of the finitely many non-zero $H^i(X)[-i]$. Each such $H^i(X)[-i]$ is in turn a finite iterated extension of shifted simples $S_a[-i]$, where $S_a[-i]$ occurs $\dim H^i(X)(a)$ times. Then $F(X)$ is a "parallel" iterated extension of various $F(S_a[-i])$, and the result follows.

We shall say that $\mathscr{D}^b(A)$ is *discrete* if for every positive element x of $K_0(A)^{(\mathbf{Z})}$ there are only finitely many isoclasses of indecomposables $X \in \mathscr{D}^b(A)$ such that $\underline{\mathrm{Dim}}\, X = x$.

PROPOSITION. *If the spectroids A and B are derived equivalent, i.e., if there exists an exact equivalence $F : \mathscr{D}^b(A) \xrightarrow{\sim} \mathscr{D}^b(B)$, and if $\mathscr{D}^b(B)$ is discrete, then so is $\mathscr{D}^b(A)$.*

This follows immediately from the estimate above. For example, let A be *derived hereditary*, i.e., derived equivalent to the (hereditary) spectroid of paths kQ of a quiver Q that is finite, directed (= without "oriented" cycle), and connected. Then $\mathscr{D}^b(A)$ is discrete iff Q is Dynkin.

1.2

We denote by $\mathscr{D}(A)_{\mathrm{perf}}$ the thick subcategory of $\mathscr{D}^b(A)$ formed by the *perfect* complexes, i.e., complexes quasi-isomorphic to bounded complexes of projectives of mod A [Gr]. Of course, $\mathscr{D}(A)_{\mathrm{perf}} = \mathscr{D}^b(A)$ iff every A-module has finite projective dimension.

We shall say that $\mathscr{D}(A)_{\mathrm{perf}}$ is *discrete* if for every positive element x of $K_0(A)^{(\mathbf{Z})}$ there are only finitely many isoclasses of indecomposables $X \in \mathscr{D}(A)_{\mathrm{perf}}$ such that $\underline{\mathrm{Dim}}\, X = x$.

1.3

The *repetitive spectroid* $\hat{A}$ [HW] associated with a finite spectroid A has as objects the pairs (a, i), where $a \in A$, $i \in \mathbf{Z}$. The morphisms $(a, i) \to (b, i)$ (resp. $(a, i) \to (b, i+1)$) are given by the elements of $A(a, b)$ (resp. the linear forms on $A(b, a)$); there is only a zero morphism $(a, i) \to (b, j)$ if $j \neq i, i+1$. Since $\hat{A}$ is self-injective, the stable category $\overline{\mathrm{mod}}\hat{A}$ associated with mod $\hat{A}$ is equipped with the structure of a triangulated category; its shift is Heller's suspension functor, and the triangles are induced by the short exact sequences of mod $\hat{A}$. There is a fully faithful exact functor E: $\mathscr{D}^b(A) \to \overline{\mathrm{mod}}\hat{A}$ which is an equivalence iff the global dimension of A is finite [H1, H2].

We shall say that $\hat{A}$ is *representation-discrete* if for every positive element u of $K_0(\hat{A})$ there are only finitely many isoclasses of indecomposables $U \in \mathrm{mod}\, \hat{A}$ such that $\underline{\dim}\, U = u$.

2. THE MAIN RESULT

2.1

A *gentle quiver* (Q, I) [AS1] is formed by a quiver Q and an ideal I of kQ with the properties (a)–(d):

(a) At every vertex of Q at most two arrows stop and at most two arrows start.

(b) I is generated by paths of length two.

(c) For every arrow β there is at most one arrow α with $\beta\alpha$ not in I and at most one arrow γ with $\gamma\beta$ not in I.

(d) For every arrow β there is at most one arrow α' with $\beta\alpha'$ in I and at most one arrow γ' with $\gamma'\beta$ in I.

THEOREM. *If A is a connected finite spectroid, the statements* (i)–(iv) *are equivalent*:

(i) *$\hat{A}$ is representation-discrete.*

(ii) *$\mathscr{D}^b(A)$ is discrete.*

(iii) *$\mathscr{D}(A)_{\mathrm{perf}}$ is discrete.*

(iv) *A either is derived hereditary of Dynkin type or admits a presentation $A \xleftarrow{\sim} kQ/I$, where (Q, I) is a gentle quiver, Q contains exactly one cycle, and the numbers of "clockwise" and of "counterclockwise" paths of length two in the cycle that belong to I are different.*

2.2. *Remarks.* (a) The classification of the derived hereditary spectroids of type **A** goes back to Assem and Happel ([AH]; see also [GR, 12.6]); they are precisely the spectroids of the form kQ/I, where (Q, I) is a finite gentle tree. The derived hereditary spectroids of type **D** were considered by Assem and Skowroński [AS2] and Keller [K]. Finally, the derived hereditary spectroids of type **E** were computed by Roggon [Ro], who found 298 + 3,341 + 21,948 = 25,587 isoclasses.

(b) According to Assem and Skowroński [AS1], the derived hereditary spectroids of type $\tilde{\mathbf{A}}$ are precisely the spectroids of the form kQ/I, where (Q, I) is a finite gentle quiver, Q contains exactly one cycle, the cycle is "non-oriented," and the numbers of "clockwise" and of "counterclockwise" paths of length two in the cycle that belong to I are *equal*.

2.3

We first concentrate on the implications (iv) ⇒ (i) ⇒ (ii) ⇒ (iii); for the less obvious implication (iii) ⇒ (iv), to be proved in Section 4, we need the cleaving method of [BGRS], which will be recalled in Section 3.

(iv) ⇒ (i). By [AS] and [PS] (see also [S]), a connected finite spectroid A admits a presentation $A \xleftarrow{\sim} kQ/I$, where (Q, I) is a gentle quiver, if and only if the associated repetitive spectroid $\hat{A}$ admits a presentation $\hat{A} \xleftarrow{\sim} k\hat{Q}/\hat{I}$, where $(\hat{Q}, \hat{I})$ is a *special biserial quiver* [SW], i.e., satisfies the conditions (a) and (c) of 2.1. In this case, the quotient $\bar{\hat{A}}$ of $\hat{A}$ by its two-sided socle is a *string spectroid*, and the indecomposables over $\bar{\hat{A}}$ (that is, the non-projective indecomposables over $\hat{A}$) are classified in terms of *strings* and *bands* [BR]: each string gives rise to one indecomposable, whereas each band gives rise to a k^*-family of homogeneous tubes of indecomposables. Now, if the gentle quiver (Q, I) satisfies the additional conditions in (iv), there are no bands for $\bar{\hat{A}}$, but arbitrarily long strings (see [Ri]). Therefore $\hat{A}$ is representation-discrete, but not locally representation-finite.

On the other hand, if A is derived hereditary of Dynkin type, $\hat{A}$ is even locally representation-finite (see [H1, V, and the references given there]).

(i) $\Rightarrow$ (ii). We observe that in $\underline{\mathrm{mod}}\hat{A}$ every $U \in \mathrm{mod}\,\hat{A}$ is isomorphic to some $U_0 \in \mathrm{mod}\,\hat{A}$ without non-zero projective summand, and $\underline{\dim}_0 U := \underline{\dim}\, U_0$ depends only on the isoclass of U in $\underline{\mathrm{mod}}\hat{A}$. If $U \to V \to W \to \Sigma U$ is a triangle of $\underline{\mathrm{mod}}\hat{A}$, then $\underline{\dim}_0 V \le \underline{\dim}_0 U + \underline{\dim}_0 W$. For Happel's fully faithful exact functor E: $\mathscr{D}^b(A) \to \underline{\mathrm{mod}}\hat{A}$ and $X \in \mathscr{D}^b(A)$ we therefore obtain as in 1.1 the estimate

$$\underline{\dim}_0 E(X) \le \sum_{i \in \mathbf{Z}} \sum_{a \in A} \dim H^i(X)(a) \underline{\dim}_0 E(S_a[-i]).$$

Consequently, if $\hat{A}$ is representation-discrete, then $\mathscr{D}^b(A)$ is discrete.

(ii) $\Rightarrow$ (iii). Trivial.

3. CLEAVING FUNCTORS

3.1

Let f: $B \to A$ be a functor between two spectroids. To avoid infinite-dimensional modules and unbounded complexes, we assume B to be *finite* and *directed* (= without "oriented" cycle in its quiver); in particular, the global dimension of B is finite. Then f induces an exact functor f_*: $\mathrm{mod}\,A \to \mathrm{mod}\,B$, $U \mapsto U \circ f$, which has a left adjoint f^*: $\mathrm{mod}\,B \to \mathrm{mod}\,A$; f^* is therefore right exact and maps a projective B-module $B(b, ?)$ to the projective A-module $A(fb, ?)$. Now f_* extends to an exact functor f_*: $\mathscr{D}^b(A) \to \mathscr{D}^b(B)$ in the obvious way. The extended f_* has as a left adjoint the exact left derived functor $\mathbf{L}f^*$: $\mathscr{D}^b(B) \to \mathscr{D}^b(A)$; it maps $\mathscr{D}^b(B) = \mathscr{D}(B)_{\mathrm{perf}}$ into $\mathscr{D}(A)_{\mathrm{perf}}$. (For the formalism of derived functors, we refer to [W].)

3.2

The functor f: $B \to A$ above is *cleaving* [BGRS, 3.1] if it satisfies the following equivalent conditions:

$(*)$ The linear maps $B(b, b') \to A(fb, fb')$ $(b, b' \in B)$ associated with f admit natural retractions.

$(**)$ The adjunction morphisms ϕ_V: $V \to f_* f^* V$ $(V \in \mathrm{mod}\,B)$ admit natural retractions.

$(***)$ The adjunction morphisms Φ_Y: $Y \to f_* \mathbf{L}f^* Y$ $(Y \in \mathscr{D}^b(B))$ admit natural retractions.

The equivalence of $(*)$ (the condition to be checked in practice) and $(**)$ is shown in [BGRS, 3.2]; that of $(**)$ and $(***)$ should be clear: If natural retractions ψ_V of the ϕ_V are given, natural retractions Ψ_Y of the Φ_Y are easily derived; conversely, if the Ψ_Y are given, we view modules as complexes concentrated in degree 0 and set $\psi_V = H^0(\Psi_V)$, observing that $\phi_V = H^0(\Phi_V)$.

We thank the referee for pointing out that cleaving functors are also known as "separable functors"; see, e.g., M. D. Rafael, Separable functors revisited, *Comm. Algebra* **18** (1990), 1445–1459.

3.3

The following result is similar to that in [BGRS, 3.1].

PROPOSITION. *Let $f\colon B \to A$ be a cleaving functor between two spectroids, where B is finite and directed. If $\mathscr{D}^b(B) = \mathscr{D}(B)_{\mathrm{perf}}$ is not discrete, then neither is $\mathscr{D}(A)_{\mathrm{perf}}$.*

Proof. Let (Y_j) be an infinite family of non-isomorphic indecomposables in $\mathscr{D}^b(B)$ with the same $\underline{\mathrm{Dim}}\, Y_j$. Then the $\underline{\mathrm{Dim}}\, \mathbf{L}f^*Y_j$ are finite in number by 1.1. If $\mathscr{D}(A)_{\mathrm{perf}}$ were discrete, the $\mathbf{L}f^*Y_j$ would give rise to only finitely many isoclasses of $\mathscr{D}(A)_{\mathrm{perf}}$ and the $f_*\mathbf{L}f^*Y_j$ to only finitely many isoclasses of $\mathscr{D}^b(B)$. But that is impossible in view of the sections $Y_j \to f_*\mathbf{L}f^*Y_j$.

4. PROOF OF THE IMPLICATION (iii) ⇒ (iv)

4.1. LEMMA. *If $\mathscr{D}(A)_{\mathrm{perf}}$ is discrete, then A is representation-finite.*

Proof. Every $U \in \mathrm{mod}\, A$ admits a minimal projective presentation $P^{-1} \to P^0 \to U \to 0$, which gives rise to a complex $P = (\dots \to 0 \to P^{-1} \to P^0 \to 0 \to \cdots)$ in $\mathscr{D}(A)_{\mathrm{perf}}$. If the dimension vector of $U = H^0(P)$ is fixed, that of $H^{-1}(P)$ is easily bounded; but then there are only finitely many possibilities (up to isomorphism) for P and hence for U. The claim follows by "Brauer–Thrall II" (see [GR, 14.8]).

4.2. LEMMA. *If $\mathscr{D}(A)_{\mathrm{perf}}$ is discrete and A is simply connected* [BoG], *then A is derived hereditary of Dynkin type.*

Although the lemma is well known, we sketch a *proof*. If the claim is wrong, we find two (simply) connected convex full subspectroids $B \supset C$ such that up to duality B is a one-point extension of C by a necessarily indecomposable C-module, B is not derived hereditary of Dynkin type, and C is derived equivalent to a hereditary spectroid C' of Dynkin type. According to Barot and Lenzing [BL], B is derived equivalent to some one-point extension B' of C' by an indecomposable C'-module, which can even be chosen to be projective. Then B is derived hereditary, but not of Dynkin type. Therefore $\mathscr{D}(B)_{\text{perf}}$ is not discrete, and neither is $\mathscr{D}(A)_{\text{perf}}$ (the inclusion $B \to A$ is cleaving).

4.3. LEMMA. *If $\mathscr{D}(A)_{\text{perf}}$ is discrete, then the endomorphism algebra $A(a,a)$ of an object $a \in A$ is isomorphic to k or to the algebra of dual numbers $k[\epsilon]$.*

Proof. Since A is representation-finite by 4.1, $A(a,a)$ is uniserial. If $A(a,a) \neq k$, $A(a,a) = k \oplus k\alpha \oplus \cdots \oplus k\alpha^{m-1}$ and $\alpha^m = 0$ for some α and some $m \geq 2$. For $n \geq m$, let A_n^m be the simply connected spectroid defined by the quiver $a_1 \xrightarrow{\alpha_1} a_2 \xrightarrow{\alpha_2} \cdots \xrightarrow{\alpha_{n-1}} a_n$ and the ideal generated by the $n-m$ paths of length m. Clearly the functor f: $A_n^m \to A$ such that $f(a_i) = a$ and $f(\alpha_i) = \alpha$ is cleaving. Therefore $\mathscr{D}(A_n^m)_{\text{perf}}$ should be discrete by 3.3, and A_n^m should be derived hereditary of Dynkin type by 4.2. We claim, however, that A_9^3, A_9^5, A_9^6, A_9^7 are *derived hereditary of type* $\tilde{\mathbf{E}}_8$, that A_8^4 *is derived hereditary of type* $\tilde{\mathbf{E}}_7$, and that *for* $m \geq 8$, A_{m+2}^m *is derived hereditary of wild type*. To verify this, the reader can easily transform the spectroids in question into hereditary spectroids of the indicated types by using suitable sequences of tilting modules [H1, III]. Our claim follows in view of the invariance of derived categories under tilting [H1, III].

Of course, the possibility that $m = 2$ cannot be excluded.

4.4. LEMMA. *If $\mathscr{D}(A)_{\text{perf}}$ is discrete and A is not simply connected, then A admits a presentation $A \xleftarrow{\sim} kQ/I$, where (Q, I) is a gentle quiver, Q contains exactly one cycle, and the numbers of "clockwise" and of "counter-clockwise" paths of length two in the cycle that belong to I are different.*

Proof. By 4.3, A does not admit a Riedtmann contour and is therefore standard [BGRS, 9]. This means that there is a Galois covering $\tilde{A} \to A$ with a non-trivial free group such that $\tilde{A}$ is simply connected, hence the filtered union of its (simply) connected convex finite full subspectroids [G, BrG]. Let B be one of them. Since the composition of the inclusion $B \to \tilde{A}$ and the covering $\tilde{A} \to A$ is cleaving, B is derived hereditary of Dynkin type by 3.3 and 4.2. We claim that B actually is of type $\mathbf{A}$. Since connected convex full subspectroids of derived hereditary spectroids of type $\mathbf{A}$ again are of the same type, we can assume that B has ≥ 9 objects. Then B is not of type $\mathbf{E}$. Suppose B is of type $\mathbf{D}$. Then we take any connected convex finite full subspectroid C of $\tilde{A}$ which contains B as well as some disjoint translate of B under the action of the Galois group. By the classification mentioned in Remark 2.2(a), C cannot be derived hereditary of Dynkin type; we obtain a contradiction and our claim is proved. We infer that $\tilde{A}$ admits a presentation by a gentle quiver, and so does A.

Let us now fix an identification $A = kQ/I$, where (Q, I) is a finite gentle quiver. As A is not simply connected, Q contains at least one cycle. If it contained two, we could construct as in [Ri] (where Ringel considers cyclic words) a quiver morphism f: $R \to Q$ where R is a "non-oriented" cycle, f maps distinct arrows of R with the same tail or the same head to distinct arrows of Q, and the numbers of "clockwise" and of "counterclockwise" paths of length two in R whose images in Q belong to I are equal. Letting J be the ideal of kR generated by the paths just mentioned, kR/J is derived hereditary of type $\tilde{\mathbf{A}}$ (Remark 2.2(b)). On the other hand, the functor $kR/J \to kQ/I$ induced by f is cleaving, a contradiction in view of 3.3. We conclude that Q contains exactly one cycle. If it is an "oriented" one, at least one of its paths of length 2 belongs to I (since kQ/I is a spectroid), and the last assertion of the lemma is true. It also holds if the cycle is "non-oriented," since otherwise kQ/I itself would be derived hereditary of type $\tilde{\mathbf{A}}$.

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